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Master thesis

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First examiner: Prof. Dr.-Ing. D. Schillinger	Supervisor: Vahid Badrkhani
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A Newton-Krylov Discontinuous Galerkin Compressible Flow Solver Using an Automatic Differentiation Derived Jacobian

Higher-order Discontinuous Galerkin (DG) discretization is applied with success in computational fluid dynamics (CFD) [1] (see Figure 1). DG discretization is attractive because it has several advantages, such as lower dispersion, more natural treatment of the convective operators, the higher extending potential for higher-order discretization, higher parallelization capacity, and better h/p refinement capability. Despite those apparent attractive advantages, the number of practical problems solved by explicit DG is very small because of convergence issues. The development of fully implicit discretization for reducing the computational cost of DG methods has been a topic of attention recently [2].

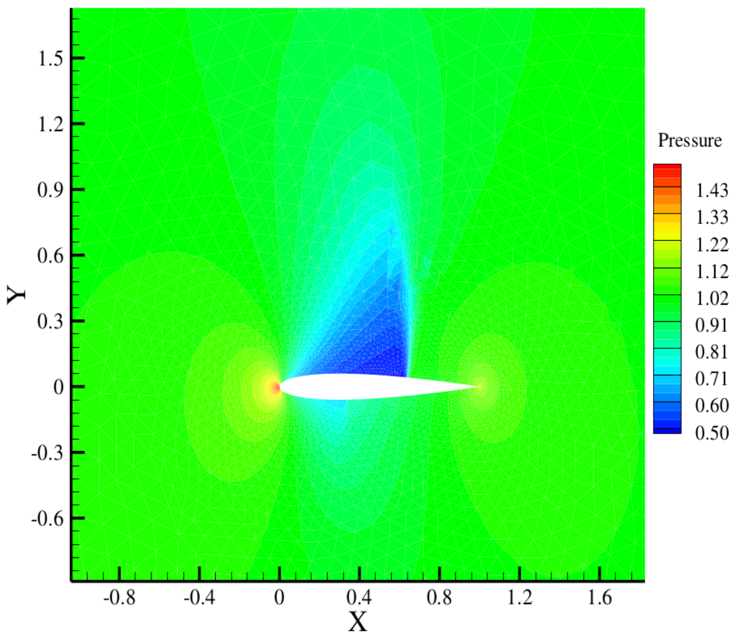

Figure 1: Transonic flow around a NACA0012 airfoil at Mach 0.8 and angle of attack of 1.25.

The Newton method is generally applied to non-linear problems which are linearized exactly. The main advantage of the Newton method is the ability to take very large time-steps and quadratic convergence when starting from a good initial guess. The linear system of equations arising from the linearization of a fully implicit scheme is solved by iterative solution methods. One of the most advanced iterative methods, the Krylov subspace technique has been employed for solving the non-linear system of equation.

In the Newton method, the Jacobian matrix is needed not only for forming the linear system but also for building the preconditioner matrix, and it is one of the most expensive parts of the implicit solver. The convergence rate of the implicit solver depends on the accuracy and correctness of the Jacobian matrix. In automatic differentiation (AD), the Jacobian can be computed automatically especially for flows with complex flux functions [3]. This method is an emerging technology for differentiating functions that enable derivatives to be computed accurately without any truncation error improving the robustness of the CFD solvers. Moreover, the AD approach was used to calculate the Jacobians of functions as accurate as exact Jacobian. The AD is used not only for residual differentiation but also for differentiation of other parameters such as derivatives of wing drag, lift and pitching moment coefficients, dissipation terms of turbulence modeling, wing shape optimization and shape optimization for fluids. The goal of this thesis is using AD derived Jacobian for DG methods.

Within the scope of this thesis, the student should study and implement different schemes for computing Jacobian (such as the AD, Matrix-Free and Finite difference method). The convergence rate and memory used of solver at different degrees and flow regimes will be analyzed and discussed. Finally, he or she should indicate how one could combine these methods to improve Newton-Krylov Discontinuous Galerkin solver.

Required knowledge includes an interest in mathematical modeling, finite element methods (preferably one has taken the DG class at IBNM) and scientific computing. In addition, programming skills with a background in discretization methods are required. This thesis will be supervised in English.

Literature:

[1] B. COCKBURN, C.-W. SHU, The local discontinuous Galerkin method for time-dependent convection–diffusion systems. SIAM Journal on Numerical Analysis, 2440–2463, 1998.

[2] D.E. KEYES, D.R. REYNOLDS, C.S. WOODWARD, Implicit solvers for large-scale nonlinear problems. Journal of Computational Physics, 2709–2722, 2006.

[3] D. PAUL, L. HOVLAND, C. MCINNES, Parallel simulation of compressible flow using automatic differentiation and PETSc, Parallel Comput., 503-519, 2001.