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Master thesis - *Masterarbeit*

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Submission of thesis until: XX.XX.20XX	Duration: 6 months
First examiner: Prof. Dr.-Ing. D. Schillinger	Supervisor: Stein Stoter
Second examiner: Prof. Dr.-Ing. U. Nackenhorst	

Essential boundary conditions for reduced basis methods

Prerequisites: a master-level class on the finite element method and the *model order reduction for computational solid mechanics* class.
This thesis will be supervised/written in English.

The standard finite element method makes use of interpolation functions that are defined on elements. Boundary conditions may be enforced by prescribing the nodal values of the interpolation functions on the associated boundary. When we make use of reduced basis methods, the potentially millions of interpolation functions are replaced by a handful 'smartly chosen' globally defined basis functions. Figure 1 shows an example mesh, and Figure 2 shows the four bases to which it may be reduced for a heat conduction model problem. These reduced bases are no-longer interpolatory, and no-longer satisfy partition of unity. Simply prescribing boundary values is thus not possible. Indeed, in the key literature (e.g. the textbooks [1,2]), the use of nonzero Dirichlet boundary conditions is not discussed. In engineering practice we often want to specify essential boundary conditions. Being able to do so for reduced basis methods is thus vital.

During this project you will focus on the following two research questions:

- How can one best incorporate nonhomogeneous (but parameter independent) essential boundary conditions in a proper orthogonal decomposition (POD) framework [1,2]?
- Can we use a variational method of weak boundary condition enforcement for parameter dependent conditions [3,4]?

To get started, we suggest a straightforward preprocessing step of the snapshot matrix in POD: by subtracting a single one of the snapshots from all other snapshots, one would obtain a single reduced basis that satisfies the boundary condition while all others satisfy homogeneous conditions. The standard approach of imposing Dirichlet conditions would again become viable.

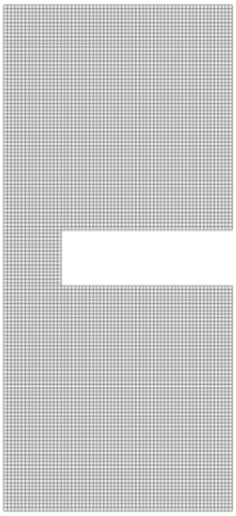

Fig 1. Mesh of 2304 elements

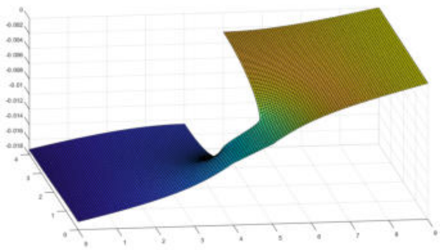

Fig 2a. First reduced basis.

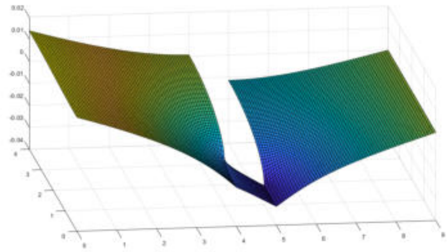

Fig 2b. Second reduced basis.

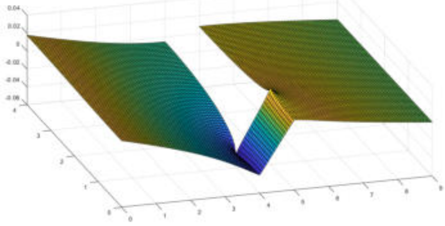

Fig 2c. Third reduced basis.

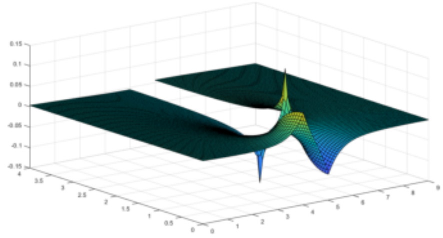

Fig 2d. Fourth reduced basis.

As a second approach, which is potentially more interesting, one could perform POD/Greedy while keeping the boundary conditions a variable in the parameter space, and use an additional term in the bilinear form that imposes the boundary conditions in the final reduced basis implementation. Examples of such approaches are Nitsche's method, penalty enforcement or the addition of a Lagrange multiplier constraint [3,4].

Literature:

[1] J.S. Hesthaven, G. Rozza and B. Stamm. *Certified reduced basis methods for parametrized partial differential equations*. Springer, 2016.

[2] A. Quarteroni, A. Manzoni and F. Negri. *Reduced basis methods for partial differential equations*. Springer, 2016.

[3] S. Fernández-Méndez and A. Huerta. Imposing essential boundary conditions in mesh-free methods. *Computer methods in applied mechanics and engineering*, 193(12-14):1257-1275, 2004.

[4] A. Embar, J. Dolbow, and I. Harari. Imposing Dirichlet boundary conditions with Nitsche's method and spline-based finite elements. *International journal for numerical methods in engineering*, 83(7):877–898, 2010.